V.

Analysis Geometrica, sive nova & vera Methodus Resolvendi, tam Problemata Geometrica, quam Arithmeticas Quæstiones. Pars prima, de Planis; Authore D. Antonio Hugone de Omerique Sanlucarense. Sold by Sam. Smith *and* Benj. Walford *at the* Prince's *in St.* Paul's Church-yard London.

THE Author of this Book being of opinion that the Method of deducing Geometric Demonstrations from an Algebraic Calculation, is forc'd and unnatural, has studied how to find an Analysis purely Geometrical, from which a Synthesis might easily be deriv'd, according to the Method of the Antients.

He begins with an Introduction consisting of about twenty Geometric Propositions; which are so many Lemmas, in order to make his Analysis the more easy; the chief Proposition of his Introduction, and which he has occasion to use most, is this: *To find two lines whose sum or difference is given, that shall be reciprocal to two given lines*; this comprehending the Construction of Quadratic Equations. He divides the rest of his Book into Four Parts. In the First he considers those Problems that are solv'd by simple Proportions. In the 2*d*. he considers those that are solv'd by using Compound Ratio. In the 3*d*. he resolves those wherein it is necessary to consider Quantities connected by the Signs + and —, And in the 4*th*. he considers Indeterminate Problems.

He Prefixes to his First Part some General Rules how to proceed in a Geometric Investigation; and because these Rules contain what is most material in his Method,

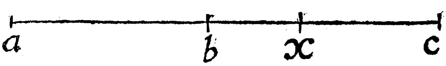
 we

we think it not improper to relate 'em as he has laid 'em down himself.

1°. An unknown Line is always terminated in an unknown Point; hence to avoid confusion, the unknown Points ought to be Denoted with the last Letters of the Alphabet *v*, *z*, *y*, *x*, &c. to distinguish 'em from the known Points *a*, *b*, *c*, *d*, &c. and if there is occasion, one and the same Point may be denoted with two Letters, when a known and unknown Line concur in it.

First Definition.

Additive Ratio is that whose Terms are dispos'd to Addition, that is, to Composition. *Subtractive Ratio* is that whose Terms are dispos'd to Subtraction, that is, to Division.

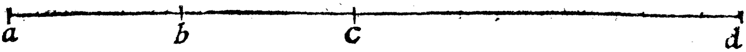


Let the Line *a c*, be divided in the Points *b*, and *x*, the Ratio between *ab*, and *bx*, is *Additive*; because the Terms *ab*, and *bx*, compose the whole *ax*; but the Ratio between *ax* and *bx* is *Subtractive*, because the Terms *ax*, and *bx*, differ by the Line *ab*.

2°. The same order of the Letters which is in the Figure, ought to be kept in your Analysis, that so by meer Inspection you may know whether the Ratio is *Additive* or *Subtractive*; and consequently whether you ought to Compose or Divide.

3°. When you are to argue by Proportions, and the Proportion lies in a Right Line, you have no other way to proceed on but by Composition or Division: Therefore if both Ratios are Additive, you must argue by Composition; if both Subtractive, by Division; so as always to use that way of arguing which is the fittest for the preservation of those Terms that are known; but when one Ratio is Additive and th'other Subtractive, the Additive must either be made Subtractive, or the Subtractive Additive; Now this change is wrought by repeating either Term.

For



For if we design to change the Additive Ratio of *ab* to *bd*, into Subtractive, let *bc* be made equal to *ab*, and thus the Ratio of *bc* to *bd*, that is, of *ab* to *bd*, will be Subtractive; and likewise, if the Subtractive Ratio of *bd* to *bc* was to be made Additive, it is but making *ab* equal to *bc*.

40. This is always to be observed, when the Terms of the Ratio which is to be reduc'd, are known; but if they are unknown, and their Sum or Difference is known, it is often convenient to use the 7*th*. and 8*th*. Proposition of the Introduction by means of which the difference of the Terms of an Additive Ratio, or the sum of the Terms of a Subtractive one, may be exprest, whence you may argue by Division or Composition. Now the 7*th*. Proposition of the Introduction is this; If a Right Line is Divided into two equal Parts, and into two unequal Parts, the middle part is the half difference of the unequal parts. The 8*th*. Proposition is this; If a Right Line is Divided into two equal parts, and a Right Line is added to it, that which is compounded of the half and of the Line added, is the half sum of the Line that is added, and of that which is compounded of the whole and the Line added.

Second Definition.

That Ratio we call Common which is Common to two Proportions whether it be Direct or Reciprocal; Let there be two Proportions *a b* : : *d*, *e*, and *b*, *c* :, : *e*, *l*, having the same Terms *b* and *e*, and constituting a Direct Ratio, this Ratio we call Common, because it is Common to both Proportions: In like manner let there be two Proportions *a*, *b* : : *e*, *l* and *b*, *c* : : *d*, *e*, each having the same Terms *b* and *e* which constitute a Reciprocal Ratio, this Ratio we call Common, because it is Common to both Proportions.

5*o*. Therefore if two Proportions have a Common Ratio, we may argue by Equality; but if a Common Ratio is wanting, it must be introduc'd, that we may proceed farther, which will be done by the Reduction of some Ratio into another equal to it.

Likewise if a Proportion lies in a Triangle or any other Figure, you must use a new Proportion by repeating some Angle, that is, by changing its Position, that so you may have two equal Terms in two different Proportions, and so may argue by Equality: Hence it is evident that, that Angle ought to be transposed, which together with the other Angles and Sides of the Figure, shews the most convenient similitude of Triangles.

6*o*. Now what is sought being assum'd as granted, all our endeavours must be to retain in arguing those magnitudes which are already known, and to extinguish as much as we can the unknown Point, and the Analyst understanding where to use Additive or Subtractive Ratio in one Proportion, and how to Introduce a Common Ratio in two Proportions, if it be wanting, will come to the end of this Resolution by necessary consequences: Now this end is obtain'd when the unknown Magnitude is found equal to some known Magnitude, or the unknown Point is in one Term, which is a 4*th.* Proportional, or in two Terms either Means or Extreams whose sum or difference is known, for a 4*th.* Proportional, or two Reciprocals will do it.

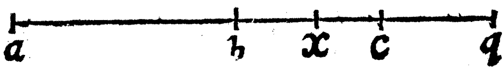
7*o*. The Analysis being ended, the order of the Construction and Demonstration is evident, for nothing else is required for the Construction, but what has, or is suppos'd to have been done in the Analysis, and for the Demonstration, nothing but to begin from the end of the Analysis and proceed to the beginning of it, observing that where the Analysis argues by Alternate or Inverted Propositions, the Synthesis argues by the same,

and

and that where the Analysis Compounds, the Synthesis Divides, and *vice versa*.

But to make those Rules more useful, it won't be amiss to shew the applications he has made of 'em in the solution of some Problems, and because there is a great variety of'em in his Book, we will chuse a few of the most remarkable as Rules in cases of the like nature.

PROBLEM.



The Line *ac* being divided at pleasure in *b* to divide it again in *x* between *b* and *c* so that *ax xc*, *bx* be proportional.

Analysis.

Let therefore	*ax*,	*xc* ::	*xc*,	*bx*.
and *Componendo*	*ac*,	*xc* ::	*bc*,	*bx*.
and *Alternando*	*ac*,	*bc* ::	*xc*,	*bx*.
Let *cq* be made = *bc*	,	*cq*		
and *Componendo*	*aq*,	*cq* ::	*bc*,	*bx*.

Therefore the Problem is solv'd.

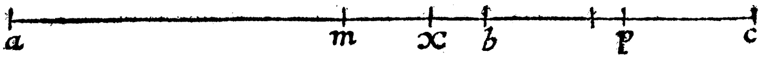
Construction.

Let the Construction be made as before.

Demonstration.

For since, by the Construction, *aq* is to *cq* as *bc* to *bx*. Therefore *Dividendo ac* is to *cq* that is to *bc*, as *xc* to *bx* and *Alternando ac* is to *xc*, as *bc* to *bx*. Therefore *Dividendo ax*, is to *xc* as *xc* to *bx*, which was to be done.

PROBLEM



PROBLEM

The Line *ac* being Divided in *b* to Divide it again in *x* between *a* and *b* so that *ax*, *xc*, *xb* be Proportional. Now becauſe in the Proportion *ax*, *xc* : : *cx*, *xb*, the firſt Ratio is *Additive* and the ſecond *Subtractive* it is evident that the *Additive* muſt either be made *Subtractive*, or the *Subtractive Additive*. But becauſe the Terms are unknown, let *ac* be biſected in *m*, and 2 *m x* will be the Difference of the Parts *ax*, *xc*; likewiſe let *bc* be biſected in *p*, and 2*xp*. will be the ſum of the Parts xc and *xb*; whence one may proceed by Compoſition or Diviſion.

Analyſis.

Let	*ax*,	*xc* : :	*xc*,	*xb*
Theref. *Componendo*	*ac*	*xc* : :	2*xp*,	*xb*
and *half. the Antecedents*	*mc*,	*xc* : :	*xp*,	*xb*
and *Convertendo*	*mc*	*mx* : :	*xp*,	*bp*

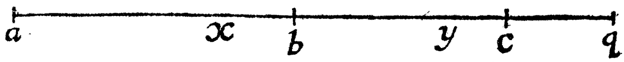
Therefore the Problem is ſolv'd. Becauſe the Point x being only in the middle Terms, we can proceed no farther. And becauſe there is nothing from whence we may infer which of the two *mx* and *xp* is the greateſt, it will be in our choice to take *mx* either for the greateſt or the leaſt part, and there will be two Solutions for which there is one Demonſtration.

Conſtruction and Demonſtration.

Let *ac* be biſected in *m* and *bc* in *p*, and to *mc* and *bp* or *pc* let two Reciprocals *mx* and *xp* be found whoſe ſum be *mp*, I ſay the thing is done.

For by the Conſtruction *mc*, *mx* : : *xp*, *bp*, Therefore *Convertendo* *mc*, *xc* : : *xp*, *xb* and doubling the Antecedents *ac*, *xc* : : 2*xp*, *xb*, but 2*xp* is the ſum of of the Terms *xc* and *xb*; therefore *Dividendo ac*, *xc* :: *xc*, *xb*, which was to be done.

PROBLEM.



To Divide the given Lines *ab bc* in *x* and *y* so that *ay* be to *xc* as *f* to *g* and *xb* to *yc* as *h* to *k*.

Conditions.

ay *xc* :: *f*, *g*
and *xb* *yc* :: *h*, *k*.

Analysis.

Let therefore *ay*, *xc* :: *f*, *g*.
and also *xb* *yc* :: *h*, *k*.
or *bc*, *cq*.

And as the sum of the Antecedents to the sum of the Consequents, so one Antecedent to its Consequent.

Therefore *xc*, *yq* :: *h* *k*.
or *g* *l*.
Therefore by Equality *ay*, *yq* :: *f*, *l*.

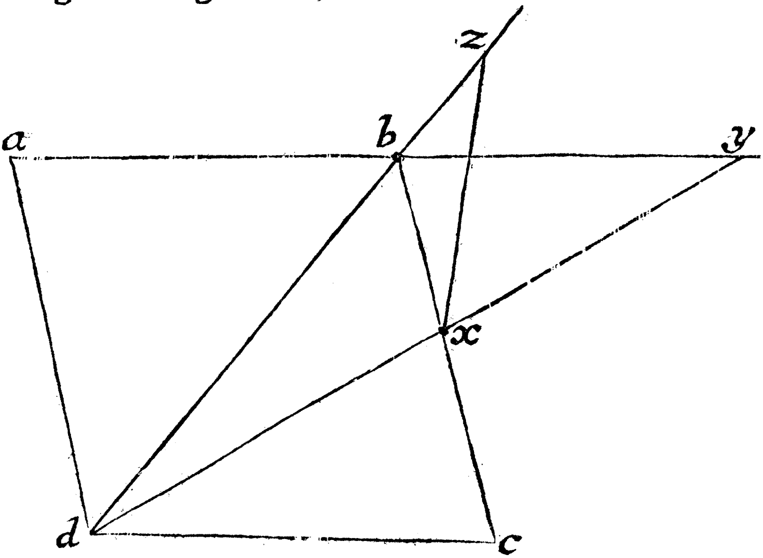
Construction and Demonstration.

Let *h* be to *k*, as *bc* to *cq*, and so *g* to *l*, Let *aq* be be Divided in *y* in the Ratio of *f* to *l*, and let *ay* be to *xc* as *f* to *g*. I say that *xb*, *yc* :: *h*, *k*. for since by the Construction *ay* *yq* :: *f*, *l*; and *ay* to *xc* as *f* to *g* : by Equality *xc* will be to *yq*, as *g* to *l* that is as *bc* to *cq* and because the difference of the Antecedents is to the difference of the Consequents, as one Antecedent to its Consequent, *xb* will be to *yc* as *bc* to *cq*, that is, as *h* to *k*, which was to be done.

PROBLEM.

A Square or Rhombus *a b c d* being given to draw

draw from the Angle *d* to the opposite side produc'd *ab* a right line *dxy*, and to make *xy* equal to a right Line given *m*.



Let therefore *xy* be equal to *m*.
by the 2*d*. of the 6*th*. Book of *Euclid* *ab*, *dy* : : *dx*, *xy*.
Let the Angle *dxz* be = *dby*.
and because the Triangles *dxz*, *dby* are Similar,
db, *by* : : *dx*, *xz*.
Therefore by Equality *db*, *ab* : : *xy*, *xz*.
But the Angle *xbz* = *dby* or *dxz*.
Therefore the Triangles *dxz*, *xbx* are Similar
Therefore *dz*, *xz* : : *xz*, *bz*.

Construction and Demonstration.

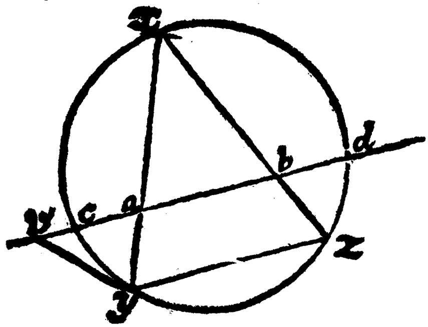
Let *db* be to *ab*, as *m* to *g*, and let *dz*, *bz* whose difference is *db* be found reciprocal to *g*. Set off from the point *z* the Line *zx* equal to *g*, and through *x* draw *dxy*, I say that *xy* is equal to the given line *m*.

For since by the Construction *dz* is to *g* as *g* to *bz*, that is *dz* is to *xz* as *xz* to *bz* :, The Triangles *dzx*, *bzx* will

will be Similar, Therefore the Angle *dxz* will be equal to the Angle *xbz*, that is, to the Angle *dby* (for the Angles *dby* and *xbz* are equal, because *dbc* in a Square or Rhombus is equal to the Angle *abd*, or its equal *ybz*, hence adding the common Angle *xby*, the Angles *dby* *xbz* will be equal.) Therefore since the Triangles *dzx*, *dby* have the Angles *dxz* and *dby* equal, and the Angle *bdx* common, they will be similar, and therefore *db* will be to *by* as *dx* to *xz* that is to *g*; but because *ad*, *bx* are parallel, *ab* will be to *by* as *dx* to *xy*. Therefore by Equality *ab* is to *db* as *g* to *xy*. But by the Construction *ab* is to *db* as *g* to *m*, Therefore *xy* is equal to *m*. Which was to be done.

PROBLEM.

A Circle *xyz* being given by Position, and two Points in it *a* and *b* being given, to draw the Lines *ax*, *xb* so that *yz* shall be Parallel to *ab*.



ANALYSIS.

Let therefore	*yz* be parallel to *ab*
Therefore the Angle	*abx* = *yzx*
Let the Angle	*ayv* be made = *abx*
Therefore the Angle	*ayv* = *yzx*
Therefore	*x*, *v*, *y*, *b*, are in a Circle
Therefore the Rectangle	*vay* = *xay*
But the Rectangle	*xay* = any Rectangle through *a*
Theref. the Rectangle	*vab* = any Rectangle through *a*.

Construction and Demonstration.

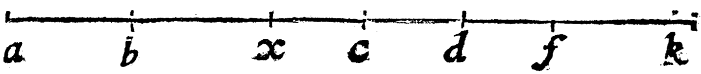
Let the Rectangle *vab* be made equal to any Rectangle through *a* such as *cad*, let the Tangent *vy* be drawn through *a* let the line *yx*, and through *b* the line *xz* be drawn, let *yz* be join'd, I say that *yz* is parallel to *ab*.

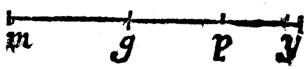
For since the Rectangle *vab* has been made equal to *cad*, and *xay* is equal to the same, the Rectangles *vab xay* will be equal: Therefore the points *x*, *v*, *y*, *b*, will be in a Circle, and the Angles *ayv*, *abx* upon the same Line *xv* will be equal, but because *vy* touches the Circle *xyz* and *xy* cuts it, the Angle *ayv* is equal to *yzx*. Therefore the Angles *yzv abx* will be equal, Therefore the Lines *yz ab* will be parallel, which was to be done.

The following Problem is taken out of the second Book.

PROBLEM.

The Line *ad* between *b* and *c* being Divided in *b* and *c*, to Divide it again in *x* so that the Rectangle *axb* be to the Rectangle *dxc* as *mp* to *gp*.





ANALYSIS.

Let therefore	axb	dxc ::	mp,	gp
Therefore if you make	ax,	xd ::	mp,	py
And also	bx,	xc ::	py	gp

The Problem will be solv'd, for the products of the Analogous Terms will restitute the Proportion.

Let therefore	ax,	xd ::	mp,	py
and *Componendo*	ax,	ad ::	mp,	my
Let *mg*, *mp*, *ad*, *ak* be proportional		ak	mg	
Let also	bx,	xc ::	py,	gp
and *Componendo*	bc,	xc ::	gy,	gp
Let *bc*, *cf*, *mg*, *gp* be proportional	cf			mg
Therefore *Componendo*	xf,	xc ::	my,	mg
and by equality	xf,	xc ::	ak,	ax
and *Convertendo*	xf,	cf ::	ak,	xk

The following Problem is taken out of the third Book.

The

PROBLEM.

The Line *ac* being divided any where in *b*, to divide it again in *x* between *b* and *c* so that the Rectangle *axb* shall be equal to the Rectangle *bxc* together with the double square of *xc*.

ANALYSIS.

Let therefore	*axb*	=	bxc + 2xcx		
But by 3. 2. El.	bcx	=	bxc + xcx		
Therefore	axb	=	bcx + xcx		
Let *cd* be made = *bc*, theref.	bcx	=	dcx		
Therefore	axb	=	dcx = + xcx		
that is by 3. 2. El.	axb	=	dxc		
Therefore	ax,	xc	::	xd,	bx
and *Componendo*	ax,	xc	::	db,	bx
Let *cf* be made = bd				cf	

and as the sum of the Antecedents, to the sum of the Consequents. So one Antecedent to its Consequent.

Therefore af, bc :: cf, bx.

Therefore the Problem is solv'd.

Construction and Demonstration.

Let *cd* and *df* be made equal to *bc*, and let *af*, *bc*, *cf*, *bx*, be proportional, I say the thing is done.

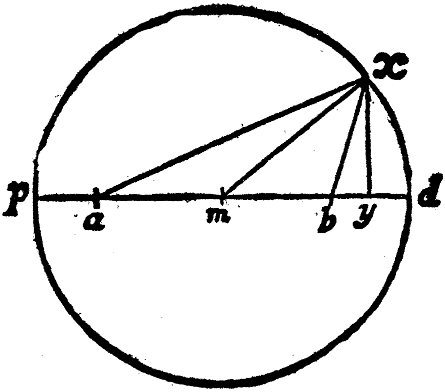
For since *af*, *bc* :: *cf*, *bx*, and the difference of the Antecedents to the difference of the Consequences as one Antecedent is to its Consequent, *ac* will be to *xc*, as *cf* or *bc* to *bx*, and the Rectangle *axb* will be equal to the Rectangle *dxc*, that is, to the Rectangle *dcx* together with the Square of *xc* or (because *bc* and *cd* are equal) to the Rectangle *bcx* with the Square of *xc*; But the Rectangle *bcx* is equal to the Rectangle *bxc* and the Square of *xc*: Therefore the Rectangle *axb* is equal to the Rectangle *bxc*, and the double Square of *xc*. Which was to be done.

The following Proposition is taken out of the 4th. *Book.*

PROBLEM.

Two Points *a* and *b* being given, to draw the two Lines

Lines *ax xb*, whose Squares together shall be equal to the Square given *gg*.



Le *axb* whose height is *xy* be the Triangle required. Bisect *ab* in *m* and draw *mx*.

ANALYSIS.

Let therefore	axa + xbx = gg
But by the 13*th*. of the Introd. *axa* + xbx	= 2ama + 2mxm
Therefore	gg = 2ama $\mp$ 2mxm
or	gg—2ama = 2mxm

Therefore the Problem is solv'd, but the Length of *mx* being given and not its Position, it is evident that it may be the Semidiameter of a Circle whose Circumference shall be the *Locus* of the point *x*.

Construction and Demonstration.

From the Square given *gg* Subtract the double Square of *am*, the Square root of half the remainder shall be the line *mx*; with the Center *m* and distance *mx*, describe the Circle *pxd*, I say that any point *x* taken in its Circumference resolves the Problem.

For since the double of the Squares of *am* and *xm* is equal to the Square *gg*, by the Construction, and by the 13*th*. Proposition of the Introduction to the Squares *ax* and *xb*: The two Squares *ax* and *xb* together will be equal to the Square *gg*. Which was to be done.

FINIS.

ERRATA.

Page 355. l. 1. for IV. r. III. p. 356. l. 26. for III. r. IV. and for *subtract*, *subtraction*, &c. r. *substract*, &c. p. 357. l. 33. r. *Sosigenes*.